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ON THE LAW OF THE VARIATION OF E-REGION MAXIMUM WITH THE ZENITH DISTANCE OF THE SUN *

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ABSTRACT. The diurnal records of E-layer critical frequency ($f^\circ E$) at Calcutta for the years 1949-52 are analysed to determine the value of n in Chapman's formula $f^\circ E = f_s (\cos\chi)^n$. n is found to vary from 0.15 to 0.50 instead of remaining constant at the Chapman value of 0.25 The sub-solar frequency f_s also undergoes similar variations. There appears to be a seasonal component of n and f_s-variations which is most marked in the years 1950 and 1951. The mean trend of these variations follows that of the 12-month running average Zürich sunspot number. The results obtained are discussed in relation to those obtained by previous workers. The departures from the Chapman law may be attributed to changes in the height distribution of O_2 in the transition region (where the atmospheric composition changes from one of O_2 and N_2 to one of O and N_2) where the E-region is formed by the ionization of O_2 Detailed study of the variations of n and f_s will be helpful in the for accurate prediction of E-region transmission conditions.

1. INTRODUCTION

According to the well known Chapman process of formation of the ionospheric layers, the maximum ionization density for a layer, under equilibrium condition, varies as $\sqrt{\cos\chi}$, where χ is the zenith distance of the sun. Records of observation show, however, that this law is obeyed only approximately for the case of the E and F_1-regions and very often not at all for the F_2-region. The subject of F_2-variation has received wide attention and many hypotheses have been put forward to explain its anomalous behaviours. The E and F_1-variations, however, appear to have received lesser attention. This is possibly because the departure of the variations from the $\sqrt{\cos\chi}$-law had been supposed to be only of minor importance. In recent years, however, the subject is being discussed by ionospheric workers and a number of investigations have been made, particularly to understand the morphology of the E and F_1-layer variations with zenith angle of the sun in differsnt seasons and latitudes, and with the phase of the solar cycle. In the present paper the results of some studies that have been made for the latitude of Calcutta on the nature of the variation of E-layer maximum with χ will be described. To understand better the significance of the results obtained a brief resumé will first be given of the investigations already made on the subject.

* Communicated by Prof. S. K. Mitra

2. A RESUME' OF CONTEMPORARY WORK ON THE SUBJECT

As mentioned in the introduction, according to Chapman's theory (Chapman, 1931) of ionized layer formation, the maximum ionization density of the layer, under equilibrium condition, varies as $\sqrt{\cos\chi}$, where χ is the solar zenith angle. Since the penetration frequency (f) of the incident wave (for a given angle of incidence) varies as the square root of the maximum ionization density, we write

$$f = f_s\,(\cos\chi)^n$$

where $n = 0.25$ and f_s is the sub-solar penetration frequency, that is, the penetration frequency when the sun is at the zenith.

The above law is found to hold generally for the E-layer except that the exponent n of $\cos\chi$ is found to be different from 0.25. The value of n that gives the best fit with the observed results can be determined conveniently by plotting $\log f$ against $\log\cos\chi$. One obtains a straight line and the slope of the line gives the value of n. The value of f_s is obtained by extrapolation of the straight line to $\chi = 0°$.

According to Tremmellen and Cox (1947) the average value of n at moderate latitudes may be taken as 0.31. They have also suggested a linear variatian of f_s, with the sunspot number R, given approximately by the expression

$$f_s = 3.3(1 + 0.00228\,R)$$

Harnischmacher (1951) has studied the month to month variations of the value of n for the stations Huancayo (12.0°S) Washington (38.7°N), Kochel (47.7°N) and Watheroo (30.3°S). He found the values to lie between 0.25 and 0.40, with a few exceptions. A seasonal variation of n was apparent at Washington with minima in winter and maxima in summer. At the other three stations no such regular variation could be observed, though, the value of n varied considerably from month to month. The value of the sub-solar penetration frequency f_s was found to be closely related to the sunspot number. It was different for the four different stations, being highest for Huancayo.

Values of n and f_s for the months of May, June and July for the years 1949 and 1950 have been derived by Scott (1952) for a large number of stations distributed throughout the globe. He found the average value of n to be about 0.33 at moderate latitudes, dropping sharply to between 0.10 and 0.20 near the north auroral zone and recovering again to 0.25 further north. Another sharp drop was found to appear at Calcutta (geomagnetic latitude 12°N).

Scott has also studied the seasonal and solar cycle dependence of n and f_s at the Canadian stations, Ottawa (45.4°N), St. John's (47.6°N), Prince Rupert

(54.3°N) and Churchill (58 8°N) n and f_s were found to rise and fall together month by month. At Ottawa and Prince Rupert a seasonal variation was found with the minima occurring regularly in winter and the maxima either at the equinoxes or in summer. An approximately linear relationship was also found between f_s at these two stations and the 12-month running average sunspot number S, given by

$$f_s = 3.2 + 5.5 \times 10^{-3} S$$

3. ANALYSIS OF CALCUTTA DATA

The data for the period, June 1949 to December 1952, as recorded at the Ionosphere Laboratory, Calcutta (22 6°N) have been analysed for the determination of the monthly values of n and f_s. The method adopted for the determination was the same as explained in Sec. 2. The hourly mean values of $f^\circ E$ for the months were plotted against the hourly mean values of log cos χ on semi-log graph. n was obtained from the slope of the curve and f_s determined by extrapolation to $\chi = 0°$. Observations for zenith angles greater than 84° were omitted, as, for such large zenith angles, the $\sqrt{\cos \chi}$-law of Chapman ceases to hold. Two typical $f^\circ E$ – log cos χ curves are shown in figure 1.

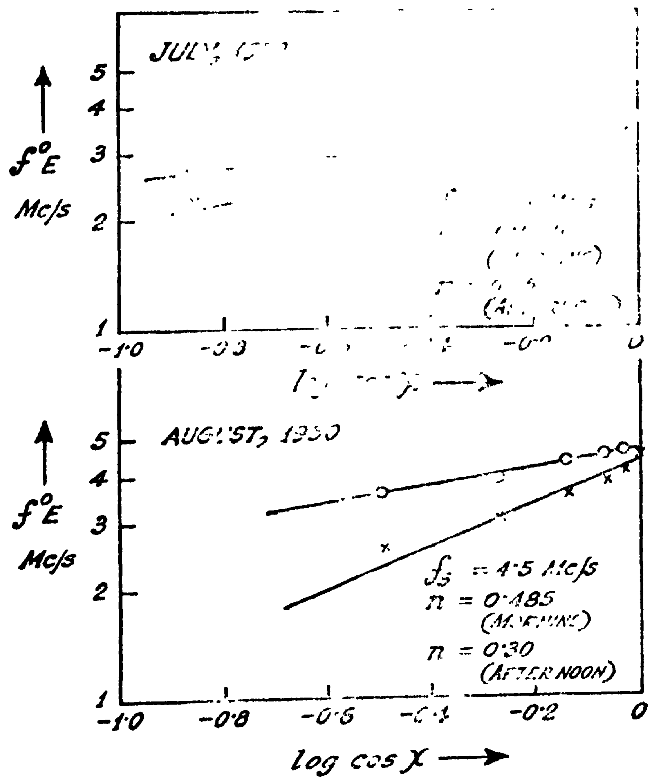

FIG. 1

Typical $f^\circ E$ vs log cos χ curves showing the relation between $f^\circ E$ and solar zenith angle χ.

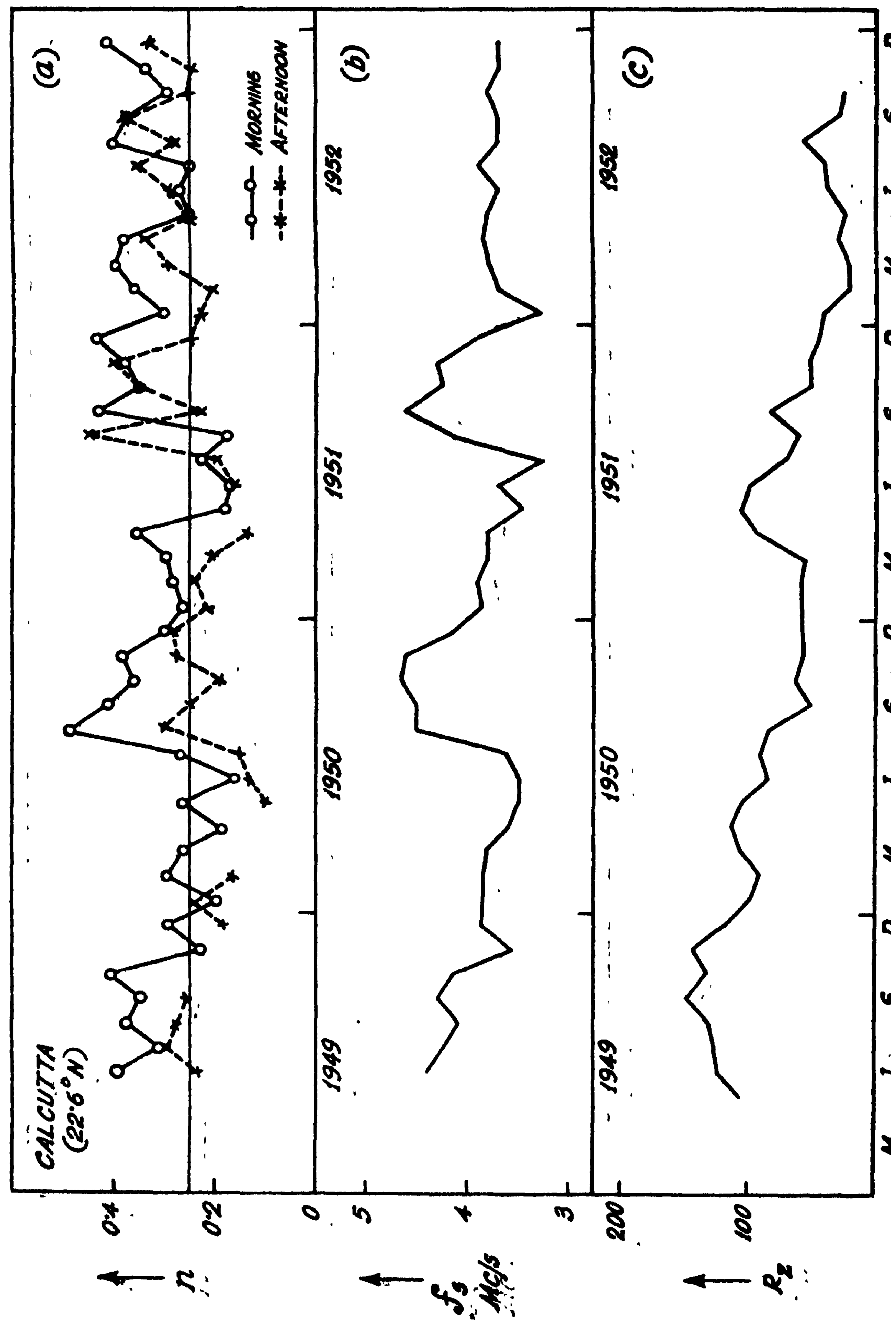

FIG. 2

The month to month variations of n and f_s at Calcutta from June 1949 to December 1952 [curves (a) and (b)]
Curve (c) shows the monthly mean values of the Zürich sunspot number (R_z).

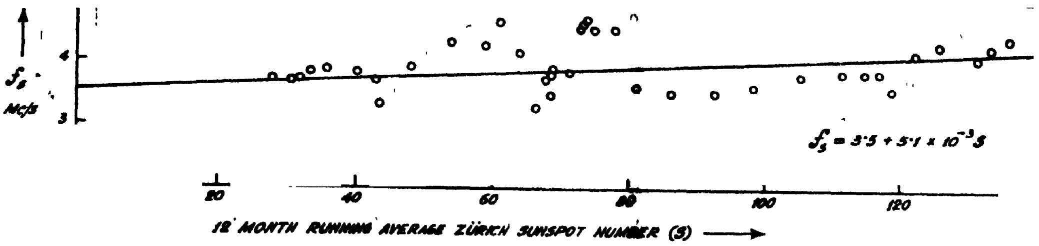

FIG. 3

The dependence of the sub-solar frequency f_s on the 12-month running average sunpot number S.

The month to month variations of n and f_s during the period under review are depicted in curves (a) and (b) in figure 2. Curve (c) shows the month to month variations of the Zürich sunspot number. It will be noted that the variations of n and f_s [curves (a) and (b)] are more or less parallel. There appears to be some sort of seasonal variation in both the n and f_s-curves, the variations being most marked for the years 1950 and 1951. A comparison of curves (b) and (c) shows, however, that what appears to be a seasonal variation of f_s may also be interpreted as a sort of rough inverse correlation between f_s and sunspot number variation. The f_s-variation has also a general tendency of following the trend of sunspot activity. In figure 3, the monthly mean values of f_s have been plotted against the 12-month running average sunspot number. The straight line giving the best fit is represented by

$$f_s = 3.5 + 5.1 \times 10^{-3} S$$

The values of f_s undergo large deviations about this mean trend.

5. DISCUSSION

The apparent seasonal variations of n and f_s at Calcutta and their parallelism, as seen in curves (a) and (b) of figure 2 are effects similar to those observed at Washington (Harnischmacher, 1951), Ottawa and Prince Rupert (Scott, 1952). However, it is to be noted that the phase of the Calcutta variations is different from that of Washington. At Washington the minima occur in winter and the maxima in summer, while at Calcutta the minima occur in summer and the maxima in autumn. Further, as already mentioned, regular seasonal variation has not been observed at all the stations studied, though, large month to month variations have been noted at Huancayo, Kochel, Watheroo (Harnischmacher, 1951). This is also confirmed by the analysis of the Singapore records by the author as depicted in figure 4. It will be seen that though the general trend of the curve follows the sunspot activity (as at other stations) there is little or no seasonal variation.

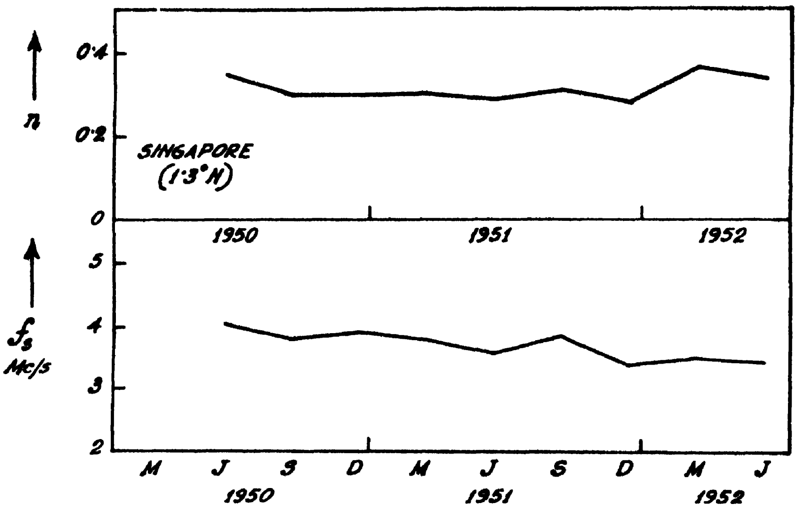

FIG. 4
The variations of n and f_s at Singapore.

One interesting fact may be mentioned in this connection. As already mentioned, Scott in his study of the latitudinal variation of n (from analysis of the May, June, July records of 1949, 1950 of a number of stations), found that there is an unexpected drop in the value of n at Calcutta in the latitude variation curve. This he has explained as due to abnormal E-ionization which is very frequent at Calcutta in summer. The explanation is, however, not tenable, as the $f^{\circ}E$ data, as recorded at Calcutta, are carefully made free from sporadic E-contamination. The result obtained by Scott is merely the effect of the apparent seasonal variation of n at Calcutta with the minima in summer.

Attempts have been made to explain the deviation of n from the Chapman value of 0.25 as due to variations in the scale height (H) and recombination co-efficient (α) in the region of the E-layer.

According to Pfister (1950), H has a maximum around noon and the diurnal variation of H may be represented by some positive power of $\cos \chi$. However, a variation of H (*i.e.*, temperature) also causes a variation in the effective recombination co efficent α, given by

$$\alpha = \alpha_0 \left[\frac{T}{T_0} \right]^r$$

where $r \simeq -1$ (Mitra and Jones, 1953). This would leave the maximum ionization in the Chapman formula

$$N_m^2 = \frac{Q}{\alpha H e} \cos \chi$$

unaltered. Pfister has also shown that the combined influence of a linear scale height gradient (H increasing with height) and a recombination co-efficient dependent only on the E-region maximum ionization would again leave N_m^2 unaffected.

Nicolet (1950) has considered α as a function of molecular concentration (α decreasing with height). His analysis shows that the combined influence of such a gradient of α and a linear gradient of scale height may lead to values of n above the normal 0.25 level. However, there are cases (*e.g.* Calcutta in summer) where n is less than 0.25. This, according to Nicolet's assumptions, is only possible if the scale height gradient becomes negative. This is improbable, since the temperature in the E-region is known to increase with height (*e.g.* rocket flight data).

Perhaps a solution of these difficulties may be found if the current theory of the E-layer formation is considered. According to this theory the E-ionization is due to ionization of molecular oxygen in the transition layer where the atmospheric composition changes rapidly from one of N_2 and O_2 to one of N_2 and O due to dissociation of O_2 by solar radiation ($\lambda \lesssim 1760$Å). The concentration of O_2 in the transition layer decreases rapidly with height, the rate of dissociation being controlled principally by the physical processes controlling the dissociation equilibrium and not by the value of H as in the case of an isothermal atmosphere following Dalton's law. E-ionization with the solar zenith angle is, therefore, not expected to follow Chapman law of variation which is deduced on the assumption that the height variation of molecular concentration follows the simple exponential law of Dalton. The fact that it does so approximately is because invetigations have shown that the decrease of O_2 concentration with height in the transition layer follows approximately an exponential law, the constant factor in the exponent then taking the place of H in Chapman's formula. The observed departures from the Chapman variation are thus traceable to the departures of the height variation of O_2 concentration from the exponential law. Further, as the value of the effective H must, of necessity, change with the solar zenith angle and with intensity of emission of the dissociating radiation from the sun, the values of n and f, must also vary correspondingly. Also, it is likely that the dissociating solar radiation producing the transition layer ($\lambda < 1760$Å) and the ionizing radiation producing the E-layer [$\lambda\lambda$ 900 – 1000Å (Nicolet, 1945) or 325 eV photons (Hoyle and Bates, 1948)] vary independently of one another with the solar activity. This will tend to destroy any close parallelism between E-ionization variation and solar activity variation which may otherwise be expected. It is also not difficult to imagine that as a result of this, coupled with the variation of recombination co-efficient with height (due to variation of temperature), the ionization may vary in an inverse manner with solar activity as it appears from curves (*b*) and (*c*) in figure 2. However, detailed investigation of these problems is beyond the scope of the present paper and is left for future study.

In conclusion, attention, is drawn to the desirability of taking into account the great variability of the value of n in predicting transmission condition *via* E-layer. At present $f^\circ E$ predictions are generally made with $n=0.25$, that is, on the assumption of $\sqrt{\cos\chi}$-law. Tremmellen and Cox (1947) have pointed out in this connection that a better value of n would be 0.31 instead of 0.25. But, in view of the abnormally high to abnormally low values (0.10 to 0.50) that n may take, assumption of a fixed value of n will lead to erroneous result. Further, the sub-solar frequency f_s, though found to follow the general trend of the solar cycle (similar to that assumed by Tremmellen and Cox and by Scott), is found to undergo large variations round the mean trend. All these factors have to be taken into account for more accurate prediction of the E-layer transmission conditions. And this will only be possible when characteristics of n and f_s variations are available over longer periods and from a larger number of stations distributed over the globe.

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